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# **Crustal density and global gravitational field models on the Moon from GRAIL and LOLA satellite data**

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# 1. Motivation:

- GRAIL (Gravity Recovery and Interior Laboratory),
- Mapping the gravitational field of the Moon in 2012,
- Altitude: ~10-90 km above the lunar surface,
- Maximum spherical harmonic degree/order (d/o): 1500,
- Spatial resolution of the gravitational field: ~3.6 km.



- ARC Discovery Project (2017 – 2019): “*Lunar crustal structure from high-res gravity, topography, and seismic data*”,
- Two geodetic/geophysical tasks important for many applications in geodesy, geophysics, and planetary sciences solved:
  - A) Determination of crustal density (inverse problem),
  - B) Calculation of global gravitational fields inferred by crustal masses (forward problem).
- Priority: application of independent mathematical methods.

## 2. Determination of crustal density:

$$\bar{C}_{n,m} = \frac{1}{R^n M(2n+1)} \int_{\Omega} \int_r \rho(r, \Omega) \bar{Y}_{n,m}(\Omega) r^{n+2} dr d\Omega$$

GRAIL-derived  
GGFM

LOLA  
topography

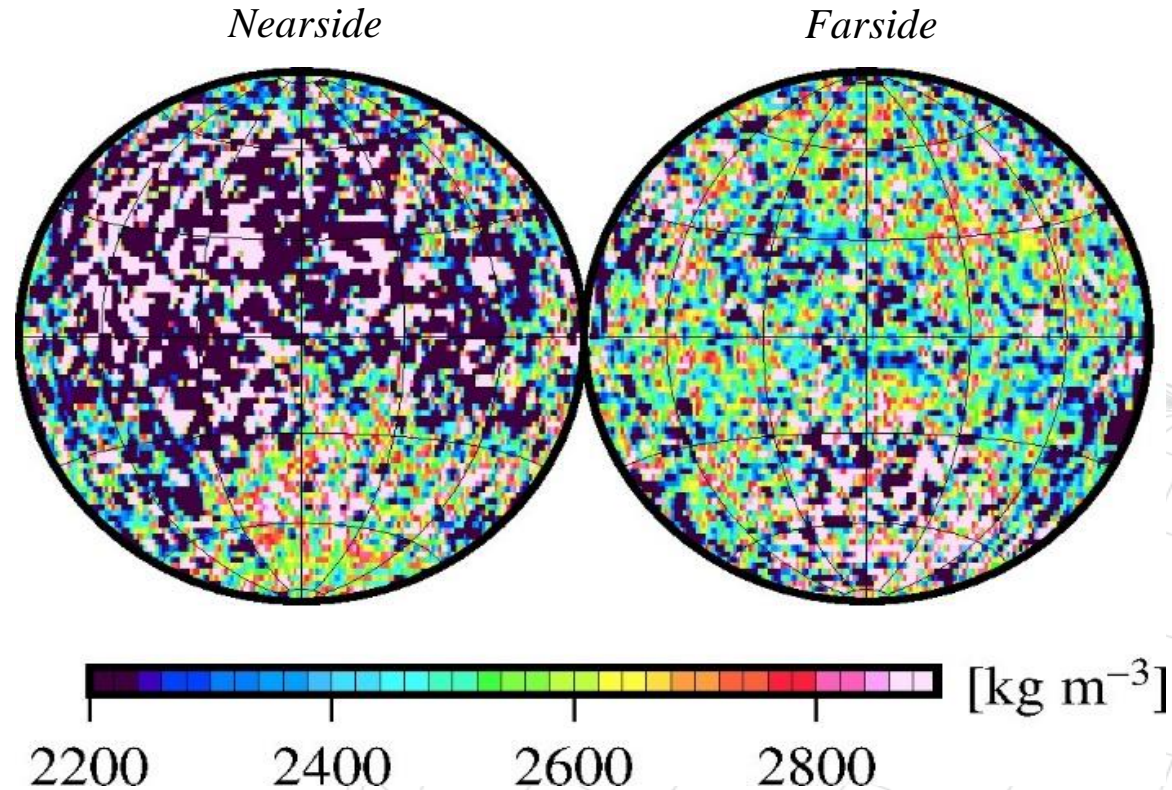
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- Mathematical model: global, spherical, and linear,
- Horizontal density variations parametrised by surface spherical harmonics,
- Crustal density estimates: 1) constant, 2) horizontally variable, and 3) spatially variable.

# Horizontally variable crustal density

*Least-squares solution:*

$$\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \hat{\mathbf{I}}$$



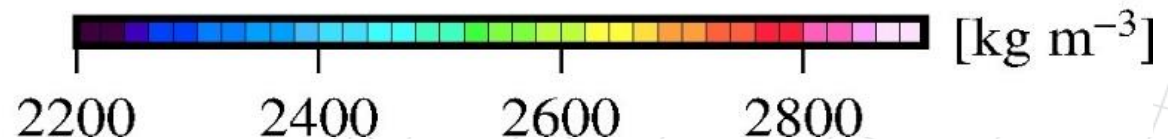
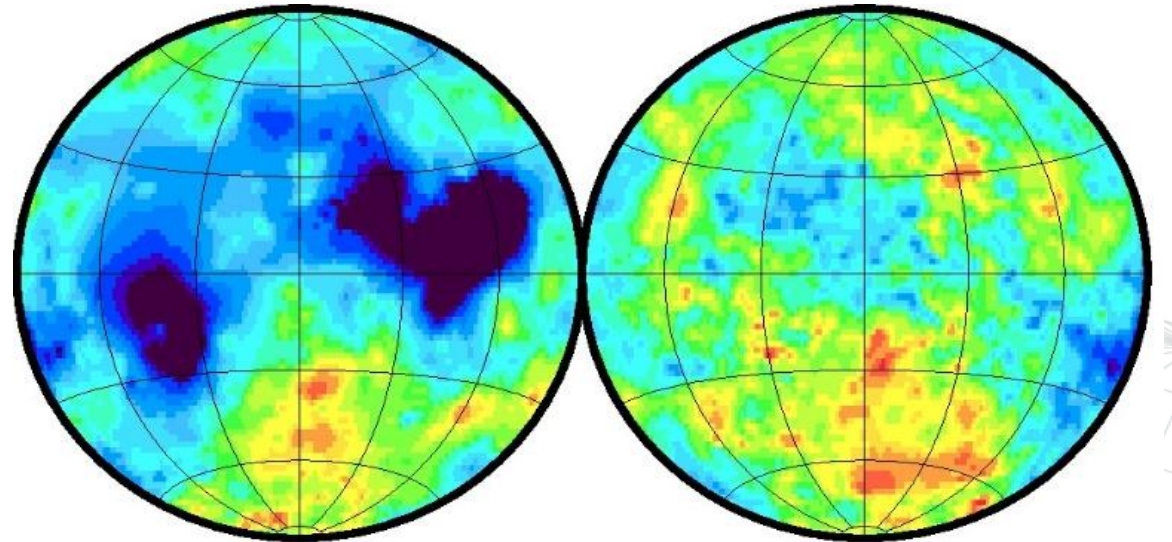
# Horizontally variable crustal density

Regularised least-squares solution:

$$\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A} + \mathbf{K})^{-1} \mathbf{A}^T \hat{\mathbf{I}}$$

Nearside

Farside



# 3. Global gravitational field models:

$$\bar{C}_{n,m} = \frac{1}{R^n M (2n + 1)} \int_{\Omega} \int_r \rho(r, \Omega) \bar{Y}_{n,m}(\Omega) r^{n+2} dr d\Omega$$

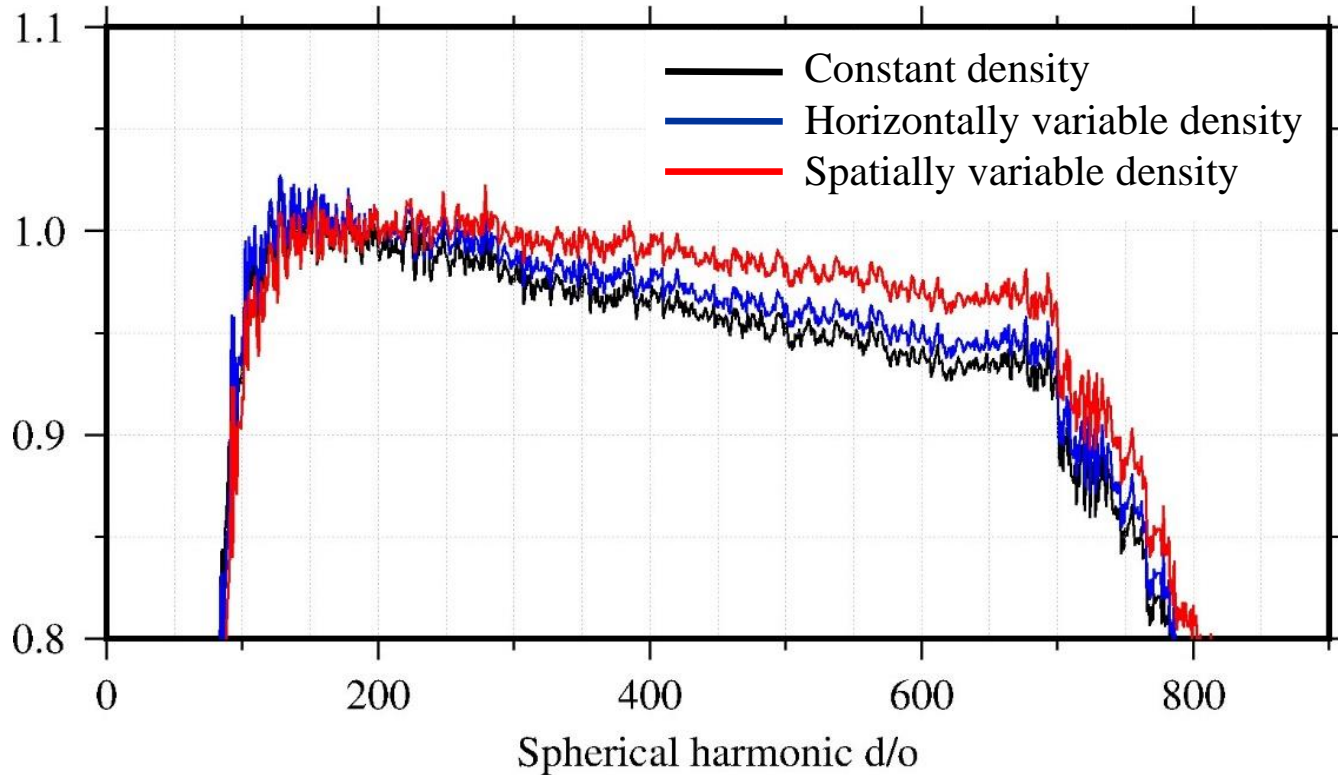
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LOLA  
topography

Our estimates

- Forward calculation of 3 GGFM's by the Rigorous Forward Modelling method (Šprlák et al. 2018),
- Maximum spherical harmonic d/o 2519 (spatial resolution ~2.2 km),
- Extensive assessment of the 3 GGFM's in spectral and spatial domains.

# Admittance (our forward GGfMs vs. GL1500E)

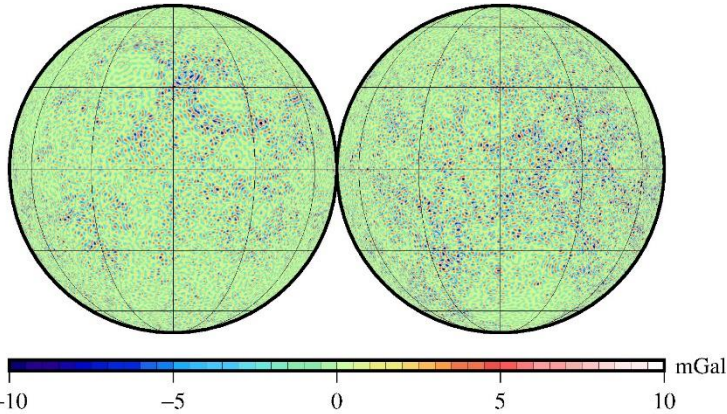




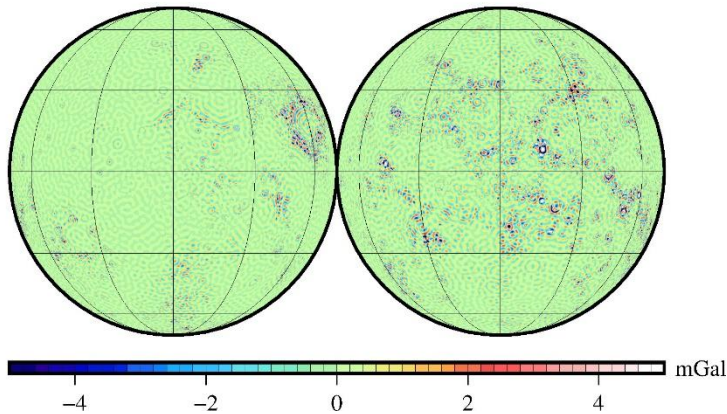
# Maps of (Bouguer) radial gravitations

*Nearside*

*Farside*

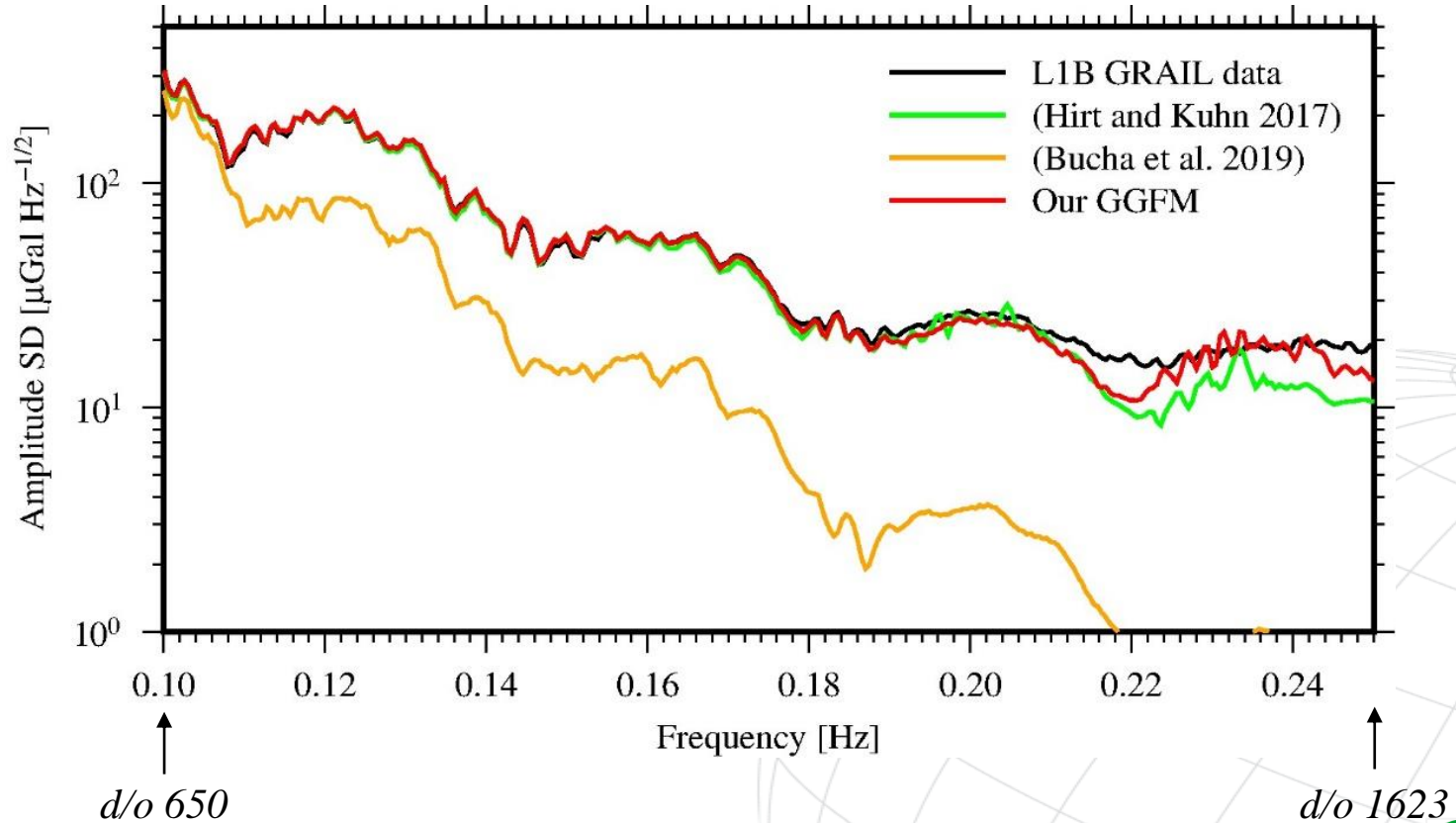


*GL1500E minus constant density*



*Spatially variable density  
vs. constant density*

# Along-track analysis (recent forward GGFM vs. L1B GRAIL)



## 4. Conclusions:


- Employment of Newton's integral in the spectral domain and solution of two geodetic/geophysical tasks,
- Formulation of a global, spherical, and linear mathematical model for crustal density estimation,
- Determination of constant, horizontally variable, and spatially variable crustal densities,
- Calculation of 3 forward GGFMs and their extensive testing in spatial and spectral domains,
- The estimated models will find applications in geodesy, geophysics, planetary sciences, navigation, etc.

# Published article:

Šprlák M, Han S-C, Featherstone W (2020) Crustal density and global gravitational field estimation of the Moon from GRAIL and LOLA satellite data. Planetary and Space Science 192:105032. <https://doi.org/10.1016/j.pss.2020.105032>.


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*Thank you for your attention!!!*

## References:

Bucha B, Hirt C, Kuhn M (2019) Divergence-free spherical harmonic gravity field modelling based on the Runge-Krarup theorem: a case study for the Moon. *Journal of Geodesy* 93(4), 489-513. <https://doi.org/10.1007/s00190-018-1177-4>.

Hirt C, Kuhn M (2017) Convergence and divergence in spherical harmonic series of the gravitational field generated by high-resolution planetary topography - A case study for the Moon. *Journal of Geophysical Research – Planets* 122(8):1727-1746. <https://doi.org/10.1002/2017JE005298>.

Šprlák M, Han S-C, Featherstone W (2018) Forward modelling of global gravity fields with 3D density structures and an application to the high-resolution (~2 km) gravity fields of the Moon. *Journal of Geodesy* 92(8):847-862. <https://doi.org/10.1007/s00190-017-1098-7>.

